

## REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

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1. REPORT DATE (DD-MM-YYYY) 05-11-2007		2. REPORT TYPE FINAL REPORT		3. DATES COVERED (From - To) From 15-02-2004 to 14-02-2007	
4. TITLE AND SUBTITLE  BASIC STUDIES IN PLASMA PHYSICS				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER FA9550-04-1-0058	
				5c. PROGRAM ELEMENT NUMBER 61102F	
				5d. PROJECT NUMBER 2301EX	
6. AUTHOR(S)  Professor Joel L. Lebowitz				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Dept of Mathematics and Physics Rutgers University New Brunswick, NJ 08903				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AF OFFICE OF SCIENTIFIC RESEARCH 875 NORTH RANDOLPH STREET ROOM 3112 ARLINGTON VA 22203 <i>Dr Robert Barker/NE</i>				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT DISTRIBUTION STATEMENT A: UNLIMITED				AFRL-SR-AR-TR-07-0480	
13. SUPPLEMENTARY NOTES					
14. ABSTRACT The primary focus of this three-year basic research effort was to obtain a better understanding of the fundamental physics of electron emission from a variety of surfaces and geometries. The very nature of emission phenomena lends itself perfectly to statistical physics analyses as used herein. The major subtopics addressed (and published) in the course of this grant include the following: space-charge-limited flow of a thin electron beam confined by a strong magnetic field, space-charge-limited flow in a rectangular geometry, ionization in a 1-D dipole model, and space-charge-limited 2-D unmagnetized flow in a wedge geometry.					
15. SUBJECT TERMS Plasma Physics, Statistical Physics, electron emission, theoretical physics, vacuum electronics					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT  Unlimited	18. NUMBER OF PAGES  14	19a. NAME OF RESPONSIBLE PERSON Prof Joel Lebowitz
a. REPORT U	b. ABSTRACT U	c. THIS PAGE U			19b. TELEPHONE NUMBER (Include area code) 732-445-3117

## FINAL REPORT

10/31/07

AFOSR GRANT FA9550-04-1-0058  
BASIC STUDIES IN PLASMA PHYSICS  
2/15/04-2/14/07

by

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### **1. Space Charge Limited Flow of a Thin Electron Beam Confined by a Strong Magnetic Field**

An approximate analytic theory was developed and implemented numerically for calculating the space charge limited current and electric field of a thin cylindrical beam or current sheet between two wide parallel electrodes. The flow is confined by a sufficiently strong magnetic field. Assuming that the potential and current density are almost homogeneous in the direction transversal to the flow we compute the beam current and profile by a variational method. We find that the average current density scales as the reciprocal of the beam width when the latter becomes very small. The total cylindrical beam current thus decreases proportionally to its diameter while the total current of a sheet becomes almost independent of the width in this regime.

### **2. Space Charge Limited Flow in a Rectangular Geometry**

We studied the spatial structure of the space charge limited current and electric field in a rectangle of arbitrary aspect ratio. The cathode and anode form two horizontal sides of the rectangle and a strong magnetic field forces the current to flow perpendicular to the electrodes. Using conformal mapping techniques we calculate the electric field outside this rectangle for any given potential distribution on its vertical boundaries. Inside the current rectangle we have a nonlinear Poisson equation with extra boundary conditions for two unknown functions: The potential and the current density. Both exhibit singular behavior at the edges of the rectangle. A semianalytic approximate method is developed for this unusual boundary value problem: We first match the boundary fields inside and outside the current region and then, using trial functions consistent with these matching conditions, we apply the least square technique and iterations to construct the solution in the current region. The analysis of the flow shows that the current wings are similar for all currents wider than one of the square cross noindent. There is also evidence that the total current does not vanish when the width goes to zero. The method of calculation appears generalizable to various geometries of vacuum and solid state devices.

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### 3. Ionization in a 1-Dimensional Dipole Model

We study the evolution of a one dimensional model atom with  $\delta$ -function binding potential, subjected to a dipole radiation field  $E(t)x$  with  $E(t)$  a  $2\pi/\omega$ -periodic real-valued function. Starting with  $\psi(x, t = 0)$  an initially localized state and  $E(t)$  a trigonometric polynomial, complete ionization occurs; the probability of finding the electron in any fixed region goes to zero. For  $\psi(x, 0)$  compactly supported and general periodic fields, we construct a resonance expansion. Each resonance is given explicitly as a Gamow vector, and is  $2\pi/\omega$  periodic in time and behaves like the exponentially growing Green's function near  $x = \pm\infty$ . The remainder is given by an asymptotic power series in  $t^{-1/2}$  with coefficients varying with  $x$ .

### 4. Excitation and Ionization of a Simple Two Level System by a Harmonic Force

A simple one-dimensional quantum system with two attractive  $\delta$ -function potentials of strength  $q$  at  $x \pm 1$  is subjected to a spatially asymmetric (as in the dipole interaction) harmonic forcing with frequency  $\omega$ . The time evolution of the system, which has two discrete energy levels  $-\omega_s < \omega_a$  (depending on  $q$ ) and a continuum spectrum, exhibits a rich dynamics including regimes where the rate of ionization becomes very small due to the 'inverse' Ramsauer effect in electron-atom collisions. The two-photon ionization with  $\omega \approx \omega_s/2$  can be enhanced when  $\omega_a = \omega_s/2$  though the one-photon ionization is not affected significantly by the location of excited level. When  $\omega$  is very close to the one or two-photon resonance the ionization rate can differ greatly from that given by the low order perturbation theory even for small forcing amplitude. This is caused in part by the fact that the dynamic Stark effect has a strong dependence on and may shift the resonance frequencies  $\omega_s, \omega_a$  up and down. When the ground state decays faster than the excited state and  $\omega$  is not close to  $\omega_s, \omega_a$ , the excited level at late times becomes and remains more populated than the ground state. The occupations of the bound states oscillate with a frequency that can be quite low compared with, in particular in the case  $\omega \approx \omega_a \approx \omega_s/2$ , but it approaches  $\omega$  when  $\omega \gg \omega_s$ . Our analysis is based on the analytic structure of the wavefunction's Laplace transform in time. It considers one- and two-'photon' processes; the higher order multiphoton processes can also be treated by our computational scheme which goes beyond the low order perturbation theory.

### 5. Regularity of the Laplace Transform and Asymptotics of the Wave Function in Time-Periodic Potentials

We study the transition to the continuum of an initially bound quantum particle in  $\mathbb{R}^d$ ,  $d = 1, 2, 3$ , subjected, for  $t \geq 0$ , to a time periodic forcing of arbitrary magnitude. The analysis is carried out for compactly supported potentials, satisfying certain auxiliary conditions. It provides complete analytic information on the time Laplace transform of the wave function. From this, comprehensive time asymptotic properties (Borel summable transseries) follow. We obtain in particular a criterion for whether the wave function gets fully delocalized (complete ionization). This criterion shows that complete ionization is generic and

provides a convenient test for particular cases. When satisfied it implies absence of discrete spectrum and resonances of the associated Floquet operator. As an illustration we show that the parametric harmonic perturbation of a potential chosen to be any nonzero multiple of the characteristic function of a measurable compact set has this property.

## 6. Transition to the Continuum of a Particle in Time-Periodic Potentials

We present new results for the transition to the continuum of an initially bound quantum particle subject to a harmonic forcing. Using rigorous exponential asymptotics methods we obtain explicit expressions, as generalized Borel summable transseries, for the probability of localization in a specified spatial region at time  $t$ . The transition to the continuum occurs for general compactly supported potentials in one dimension and our results extend easily to higher dimensional systems with spherical symmetry. This of course implies the absence of discrete spectrum of the corresponding Floquet operator.

## 7. Propagation Effects on the Breakdown of a Linear Amplifier Model: Complex-Mass Schrodinger Equation Driven by the Square of Gaussian Field

Solutions to the equation  $\partial_t \mathcal{E}(x, t) - \frac{i}{2m} \Delta \mathcal{E}(x, t) = \lambda |S(x, t)|^2 \mathcal{E}(x, t)$  are investigated, where  $S(x, t)$  is a complex Gaussian field with zero mean and specified covariance, and  $m \neq 0$  is a complex mass with  $\text{Im}(m) \geq 0$ . For real  $m$  this equation describes the backscattering of a smoothed laser beam by an optically active medium. Assuming that  $S(x, t)$  is the sum of a finite number of independent complex Gaussian random variables, we obtain an expression for the value of  $\lambda$  at which the  $q$ -th moment of  $|\mathcal{E}(x, t)|$  w.r.t. the Gaussian field  $S$  diverges. This value is found to be less or equal for all  $m \neq 0$ ,  $\text{Im}(m) \geq 0$  and  $|m| < +\infty$  than for  $|m| = +\infty$ , i.e. when the  $\Delta \mathcal{E}$  term is absent. Our solution is based on a distributional formulation of the Feynman path-integral and the Paley-Wiener theorem.

## 8. Note on a Diffraction-Amplification Problem

We investigated the solution of the equation  $\partial_t E(x, t) - iD \partial_x^2 E(x, t) = \lambda |S(x, t)|^2 E(x, t)$ , for  $x$  in a circle and  $S(x, t)$  a Gaussian stochastic field with a covariance of a particular form. It is shown that the coupling  $\lambda_c$  at which  $\langle |E| \rangle$  diverges for  $t \geq 1$  (in suitable units), is always less or equal for  $D > 0$  than  $D = 0$ . E. A. Carlen, M. C. Carvalho, R. Esposito, J. L. Lebowitz and R. Marra, Droplet Minimizers for the Cahn-Hilliard Free Energy Function We proved a theorem characterizing the minimizers in a model for condensation based on the Cahn Hilliard free energy functional. In particular, we exactly determine the critical density for droplet formation.

## 9. Phase Transitions in Equilibrium Systems: Microscopic Models and Mesoscopic Free Energies

We describe the derivation of mesoscopic free energy functionals for systems with long-range Kac potentials and analyse them in the presence of phase transitions. This yields information about the arrangement of phases in some binary



mixtures as well as explicit criteria for the stability of liquid droplets in a one-component fluid described by a Cahn Hilliard type of free energy functional.

#### 10. Ising Models with Long-Range Antiferromagnetic and Short Range Ferromagnetic Interactions

We studied the ground state of a  $d$ -dimensional Ising model with both long range (dipole-like) and nearest neighbor ferromagnetic (FM) interactions. The long range interaction is equal to  $r^{-p}$ ,  $p > d$ , while the FM interaction has strength  $J$ . If  $p > d + 1$  and  $J$  is large enough the ground state is FM, while if  $d < p \leq d + 1$  the FM state is not the ground state for any choice of  $J$ . In  $d = 1$  we show that for any  $p > 1$  the ground state has a series of transitions from an antiferromagnetic state of period 2 to  $2h$ -periodic states of blocks of sizes  $h$  with alternating sign, the size  $h$  growing when the FM interaction strength  $J$  is increased (a generalization of this result to the case  $0 < p \leq 1$  is also discussed). In  $d \geq 2$  we prove, for  $d < p \leq d + 1$ , that the dominant asymptotic behavior of the ground state energy agrees for large  $J$  with that obtained from a periodic striped state conjectured to be the true ground state. The geometry of contours in the ground state is discussed.

#### 11. Sharp Interface Motion of a Fluid Binary Mixture

We derive hydrodynamic equations describing the evolution of a binary fluid segregated into two regions, each rich in one species, which are separated (on the macroscopic scale) by a sharp interface. Our starting point is a Vlasov-Boltzmann (VB) equation describing the evolution of the one particle position and velocity distributions,  $f_i(x, v, t)$ ,  $i = 1, 2$ . The solution of the VB equation is developed in a Hilbert expansion appropriate for this system. This yields incompressible Navier-Stokes equations for the velocity field  $u$  and a jump boundary condition for the pressure across the interface. The interface, in turn, moves with a velocity given by the normal component of  $u$ .

#### 12. Large Deviations for Stochastic Model of Heat Flow

We investigate a one dimensional chain of  $2N$  harmonic oscillators in which neighboring sites have their energies redistributed randomly. The sites  $-N$  and  $N$  are in contact with thermal reservoirs at different temperature  $\tau_-$  and  $\tau_+$ . Kipnis, Marchioro, and Presutti [?] proved that this model satisfies Fourier's law and that in the hydrodynamical scaling limit, when  $N \rightarrow \infty$ , the stationary state has a linear energy density profile  $\bar{\theta}(u)$ ,  $u \in [-1, 1]$ . We derive the large deviation function  $S(\theta(u))$  for the probability of finding, in the stationary state, a profile  $\theta(u)$  different from  $\bar{\theta}(u)$ . The function  $S(\theta)$  has striking similarities to, but also large differences from, the corresponding one of the symmetric exclusion process. Like the latter it is nonlocal and satisfies a variational equation. Unlike the latter it is not convex and the Gaussian normal fluctuations are enhanced rather than suppressed compared to the local equilibrium state. We also briefly discuss more general model and find the features common in these two and other models whose  $S(\theta)$  is known.

#### 13. The Asymmetric Exclusion Process and Brownian Excursions

We consider the totally asymmetric exclusion process (TASEP) in one dimension in its maximal current phase. We show, by an exact calculation, that the non-Gaussian part of the fluctuations of density can be described in terms of the statistical properties of a Brownian excursion. Numerical simulations indicate that the description in terms of a Brownian excursion remains valid for more general one dimensional driven systems in their maximal current phase.

#### **14. Entropy of Open Lattice Systems**

We investigate the behavior of the Gibbs-Shannon entropy of the stationary nonequilibrium measure describing a one-dimensional lattice gas, of  $L$  sites, with symmetric exclusion dynamics and in contact with particle reservoirs at different densities. In the hydrodynamic scaling limit,  $L$  to infinity, the leading order ( $O(L)$ ) behavior of this entropy has been shown by Bahadoran to be that of a product measure corresponding to strict local equilibrium; we compute the first correction, which is  $O(1)$ . The computation uses a formal expansion of the entropy in terms of truncated correlation functions; for this system the  $k$ -th such correlation is shown to be  $O(L^{-k+1})$ . This entropy correction depends only on the scaled truncated pair correlation, which describes the covariance of the density field. It coincides, in the large  $L$  limit, with the corresponding correction obtained from a Gaussian measure with the same covariance.

#### **15. Fourier's Law for a Harmonic Crystal with Self-Consistent Stochastic Reservoirs**

We consider a  $d$ -dimensional harmonic crystal in contact with a stochastic Langevin type heat bath at each site. The temperatures of the "exterior" left and right heat baths are at specified values  $T_L$  and  $T_R$ , respectively, while the temperatures of the "interior" baths are chosen self-consistently so that there is no average flux of energy between them and the system in the steady state. We prove that this requirement uniquely fixes the temperatures and the self consistent system has a unique steady state. For the infinite system this state is one of local thermal equilibrium. The corresponding heat current satisfies Fourier's law with a finite positive thermal conductivity which can also be computed using the Green-Kubo formula. For the harmonic chain ( $d = 1$ ) the conductivity agrees with the expression obtained by Bolsterli, Rich and Visscher in 1970 who first studied this model. In the other limit,  $d \gg 1$ , the stationary infinite volume heat conductivity behaves as  $(l_d d)^{-1}$  where  $\lambda_d$  is the coupling to the intermediate reservoirs. We also analyze the effect of having a non-uniform distribution of the heat bath couplings. These results are proven rigorously by controlling the behavior of the correlations in the thermodynamic limit.

#### **16. Absolute Continuity of Projected SRB Measures of Coupled Arnold Cat Map Lattices**

We study a  $d$ -dimensional coupled map lattice consisting of hyperbolic toral automorphisms (Arnold cat maps) that are weakly coupled by an analytic coupling map. We construct the Sinai-Ruelle-Bowen measure for this system and study its marginals on the tori. We prove they are absolutely continuous with respect



to the Lebesgue measure if and only if the coupling satisfies a nondegeneracy condition.

### 17. On the (Boltzmann) Entropy of Nonequilibrium Systems

Boltzmann defined the entropy of a macroscopic system in a macrostate  $M$  as the log of the volume of phase space (number of microstates) corresponding to  $M$ . This agrees with the thermodynamic entropy of Clausius when  $M$  specifies the locally conserved quantities of a system in local thermal equilibrium (LTE). Here we discuss Boltzmann's entropy, involving an appropriate choice of macro-variables, for systems not in LTE. We generalize the formulas of Boltzmann for dilute gases and of Resibois for hard sphere fluids and show that for macro-variables satisfying any deterministic autonomous evolution equation arising from the microscopic dynamics the corresponding Boltzmann entropy must satisfy an  $\mathcal{H}$ -theorem.

### 18. A Random Matrix Model of Relaxation

We consider a two level system,  $S_2$ , coupled to a general  $n$  level system,  $S_n$ , via a random matrix. We derive an integral representation for the mean reduced density matrix  $\rho(t)$  of  $S_2$  in the limit  $n \rightarrow \infty$ , and we identify a model of  $S_n$  which possesses some of the properties expected for macroscopic thermal reservoirs. In particular, it yields the Gibbs form for  $\rho(\infty)$ . We consider also an analog of the van Hove limit and obtain a master equation (Markov dynamics) for the evolution of  $\rho(t)$  on an appropriate time scale.

### 19. Microscopic Origins of Irreversible Macroscopic Behavior: An overview

Time-asymmetric behavior as embodied in the second law of thermodynamics is observed in *individual macroscopic systems*. It can be understood as arising naturally from time-symmetric microscopic laws when account is taken of a) the great disparity between microscopic and macroscopic scales, b) initial conditions, and c) the fact that what we observe is "typical" behavior of real systems—not all imaginable ones. This is in accord with the ideas of Maxwell, Thomson, and Boltzmann and their natural quantum extensions. Common alternate explanations, such as those based on equating irreversible macroscopic behavior with the ergodic or mixing properties of probability distributions (ensembles) already present for chaotic dynamical systems having only a few degrees of freedom or on the impossibility of having a truly isolated system, are either unnecessary, misguided or misleading. Specific features of macroscopic evolution, such as the diffusion equation, do however depend on the dynamical instability (deterministic chaos) of trajectories of isolated macroscopic systems. Time-asymmetric behavior as embodied in the second law of thermodynamics is observed in individual macroscopic systems. It can be understood as arising naturally from time-symmetric microscopic laws when account is taken of a) the great disparity between microscopic and macroscopic scales, b) initial conditions, and c) the fact that what we observe is "typical" behavior of real systems—not all imaginable ones. This is in accord with the ideas of Maxwell, Thomson, and Boltzmann and their natural quantum extensions. Common alternate explanations, such as those based on equating irreversible macroscopic

behavior with the ergodic or mixing properties of probability distributions (ensembles) already present for chaotic dynamical systems having only a few degrees of freedom or on the impossibility of having a truly isolated system, are either unnecessary, misguided or misleading. Specific features of macroscopic evolution, such as the diffusion equation, do however depend on the dynamical instability (deterministic chaos) of trajectories of isolated macroscopic systems.

#### **20. Product Measure Steady States of Generalized Zero Range Processes**

We establish necessary and sufficient conditions for the existence of factorizable steady states of the generalized zero range process on a periodic or infinite lattice. This process allows transitions from a site  $i$  to a site  $(i + q)$  involving (a bounded number of) multiple particles with rates depending on the content of the site  $i$ , the direction  $q$  of movement, and the number of particles moving. We also show the sufficiency of a similar condition for the continuous time mass transport process, where the mass at each site and the amount transferred in each transition are continuous variables; we conjecture that this is also a necessary condition.

#### **21. Population Dynamics in Spatially Heterogeneous Systems with Drift: The Generalized Contact Process**

We investigated the time evolution and stationary states of a stochastic, spatially discrete, population model (contact process) with spatial heterogeneity and imposed drift (wind) in one- and two-dimensions. We consider in particular a situation in which space is divided into two regions: an oasis and a desert (low and high death rates). Carrying out computer simulations we find that the population in the (quasi) stationary state will be zero, localized, or delocalized, depending on the values of the drift and other parameters. The phase diagram is similar to that obtained by Nelson and coworkers from a deterministic, spatially continuous model of a bacterial population undergoing convection in a heterogeneous medium.

#### **22. Behavior of Susceptible Infected Susceptible Epidemics on Heterogeneous Networks with Saturation**

We investigated saturation effects in susceptible-infected-susceptible (SIS) models of the spread of epidemics in heterogeneous populations. The structure of interactions in the population is represented by networks with connectivity distribution  $P(k)$ , including scale-free (SF) networks with power law distributions  $P(k) \sim k^{-\gamma}$ . Considering cases where the transmission of infection between nodes depends on their connectivity, we introduce a saturation function  $C(k)$  which reduces the infection transmission rate  $\lambda$  across an edge going from a node with high connectivity  $k$ . A mean field approximation with the neglect of degree-degree correlation then leads to a finite threshold  $\lambda_c > 0$  for SF networks with  $2 < \gamma \leq 3$ . We also find, in this approximation, the fraction of infected individuals among those with degree  $k$  for  $\lambda$  close to  $\lambda_c$ . We investigate via computer simulation the contact process on a heterogeneous regular lattice and compare the results with those obtained from mean field theory with and without neglect of degree-degree correlations.



### **23. Pair Approximation of the Stochastic Susceptible-Infected-Recovered-Susceptible Epidemic Model On the Hypercubic Lattice**

We investigated the time-evolution and steady states of the stochastic susceptible-infected-recovered-susceptible(SIRS) epidemic model on one- and two- dimensional lattices. We compare the behavior of this system, obtained from computer simulations, with those obtained from the mean-field approximation(MFA) and pair-approximation(PA). The former(latter) approximates higher order moments in terms of first(second) order ones. We find that the PA gives consistently better results than the MFA. In one dimension the improvement is even qualitative.

### **24. Large Deviations for a Point Process of Bounded Variability**

We consider a one-dimensional translation invariant point process of density one with uniformly bounded variance of the number  $N_I$  of particles in any interval  $I$ . Despite this suppression of fluctuations we obtain a large deviation principle with rate function  $F(\rho) \simeq -L^{-1} \log \text{Prob}(\rho)$  for observing a macroscopic density profile  $\rho(x)$ ,  $x \in [0, 1]$ , corresponding to the coarse-grained and rescaled density of the points of the original process in an interval of length  $L$  in the limit  $L \rightarrow \infty$ .  $F(\rho)$  is not convex and is discontinuous at  $\rho \equiv 1$ , the typical profile.

### **25. Point Processes with Specified Low Order Correlation**

Given functions  $\rho_k(\mathbf{r}_1, \dots, \mathbf{r}_k)$ , defined on  $(\mathbb{R}^d)^k$  or  $(\mathbb{Z}^d)^k$  for  $k = 1, \dots, n$ , we wish to determine whether they are the first  $n$  correlation functions of some point process in  $\mathbb{R}^d$  or  $\mathbb{Z}^d$ , respectively, and if so what we can say about the process. We give partial answers to these questions and discuss some examples.

### **26. On the Realizability of Point Processes with Specified One and Two Particle Densities**

We investigated and gave partial answers to the following question: given one particle and pair density,  $\rho_1(\mathbf{r}_1)$  and  $\rho_2(\mathbf{r}_1, \mathbf{r}_2)$ ,  $\mathbf{r}_1, \mathbf{r}_2 \in \mathbb{R}^d$  (or  $\mathbb{Z}^d$ ), does there exist a point process, i.e. a probability measure on points in  $\mathbb{R}^d$  ( $\mathbb{Z}^d$ ), having these densities?

### **27. On the Construction of Particle Distribution with Specified Single and Pair Densities**

We discuss necessary conditions for the existence of probability distribution on particle configurations in  $d$ -dimensions i.e. a point process, compatible with a specified density  $\rho$  and radial distribution function  $g(\mathbf{r})$ . In  $d = 1$  we give necessary and sufficient criteria on  $\rho g(\mathbf{r})$  for the existence of such a point process of renewal (Markov) type. We prove that these conditions are satisfied for the case  $g(r) = 0, r < D$  and  $g(r) = 1, r > D$ , if and only if  $\rho D \leq e^{-1}$ : the maximum density obtainable from diluting a Poisson process. We then describe briefly necessary and sufficient conditions, valid in every dimension, for  $\rho g(r)$  to specify a determinantal point process for which all  $n$ -particle densities,  $\rho_n(\mathbf{r}_1, \dots, \mathbf{r}_n)$ , are given explicitly as determinants. We give several examples.

### **28. On the Distribution of the Wave Function for Systems in Thermal Equilibrium**

For a quantum system, a density matrix  $\rho$  that is not pure can arise, via averaging, from a distribution  $\mu$  of its wave function, a normalized vector belonging to its Hilbert space  $\mathcal{H}$ . While  $\rho$  itself does not determine a unique  $\mu$ , additional facts, such as that the system has come to thermal equilibrium, might. It is thus not unreasonable to ask, which  $\mu$ , if any, corresponds to a given thermodynamic ensemble? To answer this question we construct, for any given density matrix  $\rho$ , a natural measure on the unit sphere in  $\mathcal{H}$ , denoted  $GAP\rho$ . We do this using a suitable projection of the Gaussian measure on  $\mathcal{H}$  with covariance  $\rho$ . We establish some nice properties of  $GAP\rho$  and show that this measure arises naturally when considering macroscopic systems. In particular, we argue that it is the most appropriate choice for systems in thermal equilibrium, described by the canonical ensemble density matrix  $\rho_\beta = (1/Z) \exp(-\beta H)$ .  $GAP\rho$  may also be relevant to quantum chaos and to the stochastic evolution of open quantum systems, where distributions on  $\mathcal{H}$  are often used.

### 29. Canonical Typicality

It is well known that a system  $S$  weakly coupled to a heat bath  $B$  is described by the canonical ensemble when the composite  $S B$  is described by the microcanonical ensemble corresponding to a suitable energy shell. This is true for both classical distributions on the phase space and quantum density matrices. Here we show that a much stronger statement holds for quantum systems. Even if the state of the composite corresponds to a single wave function rather than a mixture, the reduced density matrix of the system is canonical, for the overwhelming majority of wave functions in the subspace corresponding to the energy interval encompassed by the microcanonical ensemble. This clarifies, expands, and justifies remarks made by Schrödinger in 1952.

### 30. Using Kinetic Monte Carlo Simulations to Study Phase Separation in Alloys

We review recent extensions of the kinetic Ising model used to investigate phase separation in binary alloys. Firstly, vacancies are included to model the diffusion of the atoms on the microscopic scale more realistically. These can change the coarsening rate and the coarsening mechanism. Secondly, the lattice is allowed to deform owing to the different sizes of the atoms and the resulting misfit between precipitates and matrix. The deformability of the lattice induces long-range elastic interactions between the atoms. These change the shape, orientation, and arrangement of the precipitates. The growth of the precipitates need not follow the  $R(t) \propto t^{1/3}$  law.

### 31. Stability of Solutions of Hydrodynamic Equations Describing the Scaling Limit of a Massive Piston in an Ideal Gas

We analyze the stability of stationary solutions of a singular Vlasov type hydrodynamic equation (HE). This equation was derived (under suitable assumptions) as the hydrodynamical scaling limit of the Hamiltonian evolution of a system consisting of a massive piston immersed in an ideal gas of point particles in a box. We find explicit criteria for global stability as well as a class of solutions which are linearly unstable for a dense set of parameter values. We present



evidence (but no proof) that when the mechanical system has initial conditions “close” to stationary stable solutions of the HE then it stays close to these solutions for a time which is long compared to that for which the equations have been derived. On the other hand if the initial state of the particle system is close to an unstable stationary solutions of the HE the mechanical motion follows for an extended time a perturbed solution of that equation: we find such approximate periodic solutions that are linearly stable.

### **32. Percolation in the Harmonic Crystal and Voter Model in Three Dimensions**

We investigate the site percolation transition in two strongly correlated systems in three dimensions: the massless harmonic crystal and the voter model. In the first case we start with a Gibbs measure for the potential,  $U = \frac{J}{2} \sum_{\langle x,y \rangle} (\phi(x) - \phi(y))^2$ ,  $x, y \in \mathbb{Z}^3$ ,  $J > 0$  and  $\phi(x) \in \mathbb{R}$ , a scalar height variable, and define occupation variables  $\rho_h(x) = 1, (0)$  for  $\phi(x) > h(< h)$ . The probability  $p$  of a site being occupied, is then a function of  $h$ . In the voter model we consider the stationary measure, in which each site is either occupied or empty, with probability  $p$ . In both cases the truncated pair correlation of the occupation variables,  $G(x - y)$ , decays asymptotically like  $|x - y|^{-1}$ . Using some novel Monte Carlo simulation methods and finite size scaling we find accurate values of  $p_c$  as well as the critical exponents for these systems. The latter are different from that of independent percolation in  $d = 3$ , as expected from the work of Weinrib and Halperin [WH] for the percolation transition of systems with  $G(r) \sim r^{-a}$  [A. Weinrib and B. Halperin, Phys. Rev. B 27, 413 (1983)]. In particular the correlation length exponent  $\nu$  is very close to the predicted value of 2 supporting the conjecture by WH that  $\nu = \frac{2}{a}$  is exact.

### **33. Percolation Phenomena in Low and High Density Systems**

We consider the 2D quenched-disordered  $q$ -state Potts ferromagnets and show that at self-dual points any amalgamation of  $q - 1$  species will fail to percolate despite an overall (high) density of  $1 - q^{-1}$ . Further, in the dilute bond version of these systems, if the system is just above threshold, then throughout the low temperature phase there is percolation of a single species despite a correspondingly small density. Finally, we demonstrate both phenomena in a single model by considering a “perturbation” of the dilute model that has a self-dual point. We also demonstrate that these phenomena occur, by a similar mechanism, in a simple coloring model invented by O. Häggström.

### **34. Space Charge Limited Two-dimensional Unmagnetized Flow in a Wedge Geometry**

This paper studies the space charge limited current in an infinite wedge geometry in two dimensions. This geometry permits a reduction of the problem to a set of easily solved ordinary differential equations. The system, though very simplified, exhibits features similar to those expected to occur in many realistic systems with inhomogeneous electric fields. We obtain, in particular, a universal form for the particle trajectories and a non-monotone charge distribution with accumulation at both the cathode and the anode. The explicit solution of the

model can be useful for testing numerical schemes. The case of a very low density current is also considered. Relaxation of the geometrical limitations of the model are studied using conformal mapping techniques. Possible applications to realistic systems, which can be tested by simple experiments, are presented.

#### List of Publications

Supported by AFOSR GRANT FA FA9550-04-1-0058, 2/15/04 - 2/14/07

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